$$A \xhookrightarrow{i} A\langle X\rangle \xrightarrow{\rho} \mathrm{End}(V)$$

$$\mathrm{Res}_A^{A\langle X\rangle} : A\langle X\rangle\text{-}\underline{\mathrm{Mod}} \longrightarrow A\text{-}\underline{\mathrm{Mod}}$$

$$(V, \rho) \longmapsto (V, \rho \circ i)$$

$$\mathrm{Hom}_A\left(M, \mathrm{Res}_A^{A\langle X\rangle} N\right) \cong \mathrm{Hom}_{A\langle X\rangle}(?M, N)$$

$$\varphi \longmapsto \tilde{\varphi}$$

Try: $X.m = mX \in M[X]$

$$(Xa).m = X.(a.m) = (a.m)X$$

$$(aX).m = a.(X.m) = a.(mX) = (a.m)X$$

$M\langle X\rangle = m_1 \amalg m_2 \amalg \cdots \amalg m_n$?

Note $\mathrm{Hom}_A(M, \mathrm{Res}^{A[X]}_A N) \cong \mathrm{Hom}_{A[X]}(M[X], N)$

$M \longmapsto M[X]$ LEFT ADJOINT TO $\mathrm{Res}^{A[X]}_A$.

$$\delta : X \longrightarrow F_{\Bbbk}(X)$$

$$\rightsquigarrow \delta : F_{\Bbbk}(X) \longrightarrow F_{\Bbbk}(X)$$

DEF A MAP $\delta : A \to M$, $\begin{cases} A & \text{ALGEBRA,} \\ M & (A,A)\text{-BIMODULE.} \end{cases}$

IS A DERIVATION IF

$$\delta(ab) = \delta(a)\cdot b + a\cdot\delta(b) \quad \forall a,b \in A$$

$$\delta : X \longrightarrow F_{\Bbbk}(X)/_{I}$$

$$\delta : F_k(X) \longrightarrow F_k(X) \longrightarrow F_k(X)/I$$

$\delta : A \to M$ DERIVATION

$\varphi : M \to N$ (A,A)-BIMODULE MAP

$$(\varphi \circ \delta)(ab) = \varphi\big(\delta(a)\cdot b + a\cdot\delta(b)\big)$$
$$= (\varphi\circ\delta)(a)\cdot b + a\cdot(\varphi\circ\delta)(b)$$

EX

$\delta : X \to F_k(X) \rightsquigarrow \delta : F_k(X) \to F_k(X)$

$\varphi : F_k(X) \to F_k(X)/I$ IS AN $(F_k(X), F_k(X))$-BIMODULE MAP

EARLY 1990's: V. BAVULA, ROSENBERG, D. JORDAN

GENERALIZED WEYL ALGEBRAS. (GWAs)

R ALGEBRA ("THE BASE ALGEBRA")

$\sigma : R \longrightarrow R$ AN ALGEBRA AUTOMORPHISM.

$t \in R$

THE GWA ASSOCIATED TO (R, σ, t) IS DEFINED AS $R\langle X, Y\rangle / I$ WHERE I IS GENERATED BY $(r \cdot Y - Y \cdot \sigma(r))$

$$X \cdot r - \sigma(r) \cdot X \;, \quad Y \cdot r - \sigma^{-1}(r) \cdot Y \quad \forall r \in R$$

$$YX - t \;, \quad XY - \sigma(t) \;.$$

NOTATION: $R(\sigma, t)$

EX. $R = k[u]$ POL. ALG.

$\sigma(u) = u - 1$ ($\sigma : \{u\} \to k[u] \dots$)

($\Rightarrow \sigma(p(u)) = p(u-1)$)

$t = u$

CLAIM: $R(\sigma, t) \cong A_1(k) = \dfrac{k\langle x, \partial \rangle}{\langle \partial x - x\partial - 1 \rangle}$

$\varphi : \{x, \partial\} \longrightarrow R(\sigma, t)$

$x \longmapsto X$

$\partial \longmapsto Y$

$\rightsquigarrow \tilde{\varphi} : k\langle x, \partial \rangle \longrightarrow R(\sigma, t)$

$\tilde{\varphi}(\partial x - x\partial - 1) \equiv \tilde{\varphi}(\partial)\tilde{\varphi}(x) - \tilde{\varphi}(x)\tilde{\varphi}(\partial) - 1$

$= YX - XY - 1 = t - \sigma(t) - 1 = 0$

WE GET $\Phi : A_1(k) \longrightarrow R(G,t)$ ALG MAP.

Φ SURJ: $X = \Phi(x)$

$Y = \Phi(\partial)$

$u = t = YX = \Phi(\partial x)$

PROBLEM: PROVE THAT $A_1(k)$ IS A SIMPLE ALGEBRA (THE ONLY 2-SIDED IDEALS ARE 0 & $A_1(k)$).

[ASSUME $I \neq 0$, USE $[\partial, x^n] = nx^{n-1}$, ..]

COR. ANY NONZERO ALGEBRA MAP $\psi : A_1(k) \to B$ IS INJECTIVE. $\Rightarrow \Phi$ ISOM.